An Algebraic Approach to Image Schemas for Geographic Space

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Abstract. Formal models of geographic space should support reasoning about its static and dynamic properties, its objects, their behaviors, and the relationships between them. Image schemas, used to embody spatiotemporal experiential abstractions, capture high-level perceptual concepts but do not have generally accepted formalizations. This paper provides a method for formally representing topological and physical image schemas using Milner's bigraphical models. Bigraphs, capable of independently representing mobile locality and connectivity, provide formal algebraic specifications of geographic environments enhanced by intuitive visual representations. Using examples from a built environment, we define topological schemas CONTAINER and LINK as static bigraph components, dynamic schemas INTO and LINKTO as rule-based changes in static components, and more complex schemas REMOVAL_OF_RESTRAINT and BLOCKAGE with sequences of rules. Finally, we demonstrate that bigraphs can be used to describe scenes with incomplete information, and that we can adjust the granularity of scenes by using bigraph composition to provide additional context.

Keywords: Image Schemas, Spatial Relations, Built Environments, Bigraph Models.

1 Introduction

Modern information systems require models that incorporate notions of physical space in ways that move beyond the use of building information models (Laiserin 2002), geographic maps, and wayfinding tools for assisting human navigation though these spaces. Ubiquitous computing environments (Weiser 1993), such as ambient intelligent systems, require software agents that can detect and cause changes in physical environments. Conversely, humans and human aides (e.g., smart phones) in physical space can access and change information objects that exist in virtual space. An additional complication is that bounded regions in both physical and virtual spaces can have either tangible borders (e.g., brick and mortar walls) or borders established by fiat (e.g., the intangible dividing lines between co-worker workspaces in a shared unpartitioned office). In virtual environments systems may also be tangibly bounded (e.g., the confines of a single hard drive) or bounded by an internal software partition or firewall. Communication links in either kind of space can often cross boundaries freely. Although these issues are not new, this hybrid virtual-physical space of interaction continues to provide modeling challenges for spatial information theory.

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Image schemas (Johnson 1987), used to embody spatiotemporal experiential abstractions, model conceptual patterns that can be physical or non-physical but do not have generally accepted formalizations despite numerous attempts (Egenhofer and Rodriguez 1999, Frank and Raubal 1999, Raubal, Egenhofer, Pfoser, and Tryfona 1997, Raubal and Worboys 1999). They have a rich history as a support for spatial reasoning and inference, particularly in the use of translating spatial prepositions in language to relations (Mark 1989, Freundschuh and Sharma 1996, Frank 1998), as a basis for describing geographic scenes (Mark and Frank 1989), and as fundamental theories underlying good user interfaces and query languages (Mark 1989, Kuhn and Frank 1991, Kuhn 1993). However, distinct differences exist in the use of schemas in small-scale and large-scale environments (Frank 1998, Frank and Raubal 1998, Egenhofer and Rodriguez 1999), due in part to changes in inference patterns for spatial relations when the scale changes.

Milner (2009) argues that models for spatially-rich systems should provide visualization tools that are tightly coupled with a formal system in order to support the needs of diverse communities including end-users, programmers, system designers, and theoretical analysts. Milner's bigraphical models (Milner 2001) provide a formal method for independently specifying mobile connectivity and mobile locality. Combined with a set of reaction rules that dictate appropriate system transformations, bigraphs provide a unified platform for designing, formally modeling, analyzing, and visualizing ubiquitous systems. Bigraphs were developed for the virtual world of communicating processes and ambient information objects. While it has been argued that agents can be physical and can influence informatics domains, it has not yet been demonstrated that bigraphs are suitable models for mixed virtual and physical environments.

This paper presents an approach for using bigraphs to formally model image schemas for use in built environments. We argue that realizing key image schemas in bigraphs provides a novel and useful means to represent and visualize the static and dynamic relationships and behaviors of entities in built environments. This work arises out of research on a theoretical framework for formally modeling ubiquitous computing environments as part of the multinational Indoor Spatial Awareness Project[1], and is a step towards our goal of developing a more comprehensive spatial theory for built environments.

In the remainder of the paper we provide the background of image schemas and bigraphical models and propose the use of bigraghs to formally represent and visualize key static and dynamic image schemas in built environments. We show how bigraphs can represent scenes with incomplete information, how bigraph composition can be used to increase and decrease the granularity of scenes by providing additional context, present a built environment example utilizing image schemas and granularity shifts, and finally present our conclusions and future work.

2 Background

2.1 Image Schemas

Image schemas are abstractions of spatiotemporal perceptual patterns. In his survey Oakley (2007) describes them as “condensed redescriptions of perceptual experience

[1] http://u-indoor.org/

for the purpose of mapping spatial structure onto conceptual structure". Johnson (1987) stated that these patterns "emerge as meaningful structures for us chiefly at the level of our bodily movements through space, our manipulation of objects, and our perceptual interactions" and that schemas can also be applied to "events, states, and abstract entities interpreted as spatially bounded entities". Since our goal is to model the locality and connectivity of entities in built environments, here follow several of the more useful topological and physical schema.

2.1.1 CONTAINER

This schema associates an entity serving as a container with expected relationships and behaviors. These can be either static or dynamic, for example we often associate the spatial relations *inside* and *outside* with some physical entity (e.g., in a room) or conceptual entity (e.g., in a conversation) as well as the dynamic behavior of moving *out-of* or *into* the entity serving as the container. There is also typically an explicit or implicit second entity that participates in the relationship or behavior associated with the container. This schema is often associated with either SURFACE (Rodriguez and Egenhofer 1997) or SUPPORT (Kuhn 2007). However, many place relations, actions and behaviors in built environments can be explained with CONTAINER alone.

2.1.2 LINK

The basic LINK schema (Johnson 1987) consists of two entities connected by a "bonding structure" which could be physical, spatial, temporal, causal, or functional. He also identified potential extensions to this basic schema including relationships between more than two entities or between entities that are spatially or temporally discontiguous. When the link structure is directed, this schema can become a building block for more complex schemas such as PATH (Kuhn 2007).

2.1.3 FORCE

Johnson identified seven basic FORCE schemas, two of which, BLOCKAGE and REMOVAL_OF_RESTRAINT, are particularly relevant. BLOCKAGE involves a force vector encountering a barrier and taking any number of possible directions in response, including the removal of the barrier. REMOVAL_OF_RESTRAINT involves the actions of the force vector once the barrier has been removed by another.

2.1.4 Spatial Relations from Schemas

Many researchers have expanded upon Johnson's original concepts to incorporate aspects of importance for GIS such as spatial cognition and relations (Freundschuh and Sharma 1996, Raubal, Egenhofer, Pfoser, and Tryfona 1997, and Frank and Raubal 1999). For example, Lakoff and Nunez (2000) expand upon Johnson's schemas to derive conceptual schemas for common spatial relations, such as *in*, *out*. Given a CONTAINER schemas where the entity serving as the container has an associated interior, exterior, and boundary, IN and OUT schemas can be defined where the focus of the container is either on its interior (for IN) or exterior (for OUT).

2.1.5 Algebras for Image Schemas

Egenhofer and Rodríguez (1999) defined a CONTAINER-SURFACE algebra for reasoning with inferences about the spatial relations in/out and on/off. They argue that these schemas are particularly appropriate for small-scale "Room Space", but demonstrate that there are problems when applying it to large-scale geographic objects and spaces, or mixed environments. In particular, they note that some inference-based reasoning does not directly translate between scales (Rodrígues and Egenhofer 2000).

Kuhn (2007) proposed an algebraic theory built on image schemas for an ontological specification of spatial categories. He focused on topological (e.g., CONTAINMENT, LINK, PATH) and physical (e.g., SUPPORT, BLOCKAGE) schemas, and provided algebraic specifications that associate type classes with universal ontological categories and type membership with universal class instantiation. His model offers a powerful formalism for combining schema behaviors using type class subsumption. Our approach, which formalizes schemas using an algebraic approach, should complement his theory.

2.2 Bigraphical Models

Milner's bigraphs (Milner 2001) provide a formal method for independently specifying mobile connectivity and mobile locality, and are intended to provide an intuitive formal representation of both virtual and physical systems. Bigraphs have a formal definition in category theory (specifically as abstract structures in strict symmetric monoidal categories), but in this paper we use Milner's simpler visual descriptions that are tightly coupled with the underlying algebra. Bigraphs originate in process calculi, especially the calculus of mobile ambients (Cardelli and Gordon 2000) for modeling spatial configurations, and the Pi-calculus (Milner 1999) for modeling connectivity. Ambients, represented as nodes in bigraphs, were originally defined as "bounded places where computation occurs" (Cardelli 1999). In bigraphs nodes have a more general interpretation as bounded physical or virtual entities or regions that can contain other entities or regions. For example, a built environment can be modeled with a bigraph containing nodes representing agents, computers, rooms, and buildings that also represents the connectivity between agents and computers (Milner 2008).

2.2.1 Place Graphs

Containment relations between nodes in bigraphs are visualized by letting nodes contain other nodes. Other spatial relationships between regions such as overlap, meet, or equals are not expressible as place relations. Every bigraph B has an associated place graph B^P, which shows only the containment relations between entities. Figure 1 shows a simple bigraph B of an agent A_1 and a computer C_1 in a room R_1 with its place graph B^P (a tree). The agent is not connected to the computer.

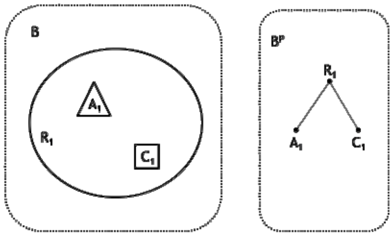

Fig. 1. Bigraph B and place graph B^P

2.2.2 Link Graphs

Connectivity, or linking relations, are represented as hypergraphs, generalizations of graphs in which an edge may join any number of nodes. Each node has a fixed number of *ports* indicating the number of links that it is permitted. Each edge represents a particular connection (relation) between the nodes it links, but does not typically denote a spatial relation such as physical adjacency. Every bigraph B has an associated link graph B^L which shows only the connectivity between nodes. Figure 2 shows the bigraph B for the simple scene when the agent A_1 is connected to the computer C_1. Both the agent and the computer now have one port each that supports the link. The place graph B^P shows only the containment relations in the scene and the link graph B^L shows only the linking relations in the scene.

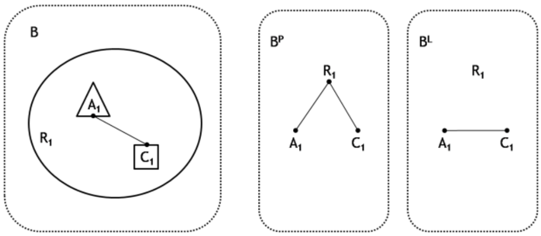

Fig. 2. Bigraph B with place and link graphs

Figure 3, a variant of Milner's example (Milner 2008), shows a more complicated scene with additional agents and an extra room and building.

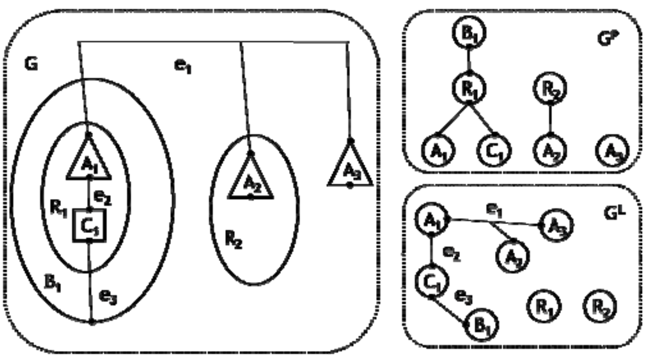

Fig. 3. Built environment G with place and link graphs

Bigraph G describes a scene in which agent A_1 and computer C_1 are inside room R_1 in building B_1, another agent (A_2) is inside a second room (R_2), and a third agent (A_3) is elsewhere. This place configuration is reflected in the place graph G^P. Three agents participate in a conference call (e_1), the first agent is on a computer (e_2), and the computer is connected to a LAN in the building (e_3). This link configuration is reflected in the link graph G^L. In this example rooms have no ports, but buildings have one for a LAN. Agents and computers each have two ports so that any agent can simultaneously connect to a conference call and to a computer. Any computer can simultaneously connect to an agent and network.

3 Image Schemas in Bigraphs

When considering a domain in which the location and connectivity of entities are the most important aspects of the setting, topological spatial schemas for CONTAINMENT and LINK can be used to describe key static topological relationships, and sequences of bigraph transformations can be used to represent changes in connectivity and locality. More complex behaviors that use the FORCE schemas of BLOCKAGE and REMOVAL_OF_RESTRAINT can be used to model scenes in which, for example, an agent has to enter a locked room or building.

3.1 Static Image Schemas in Bigraphs

Bigraphs directly support the visualization of two static spatial schemas, CONTAINMENT and LINK. A node can contain another node (IN) or be connected to another node (LINK). Examples of both schemas are found in Figures 1-3 such as an agent being IN a room or having a LINKTO a computer. These correspondences follow directly from Milner's discussions of his basic bigraph model (Milner 2008).

3.1.1 Reaction Rules for Static Image Schemas

Actions such as going *into* or *out of* a node or *linking/unlinking* can be described using bigraph reaction rules, pairs of bigraph parts indicating permissible atomic changes in bigraphs. For example, the action of an agent logging off a computer can be represented by an UNLINK_FROM rule. Figure 4 shows the visualization of the rule (left) and an example of its application to simple bigraph B to produce a new bigraph B'. No change in containment occurs when applying this rule.

This is only one example of an unlink rule. If we needed a rule to permit agents to leave a conference call we would need a new rule that permitted links between agents to be broken. Another useful rule is one that represents an agent's ability to leave a room. For this we need a reaction rule OUT_OF between an agent and a room. Since the room can contain other entities, our general rule should account for that by

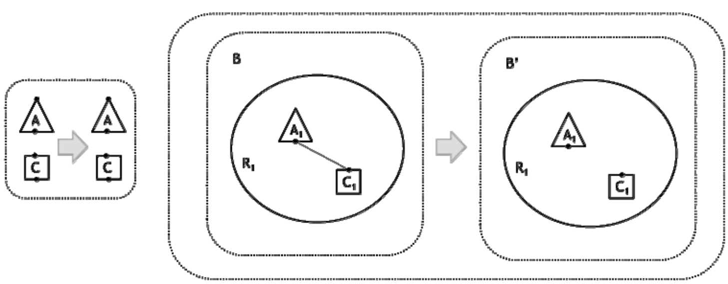

Fig. 4. UNLINK_FROM rule applied to bigraph B

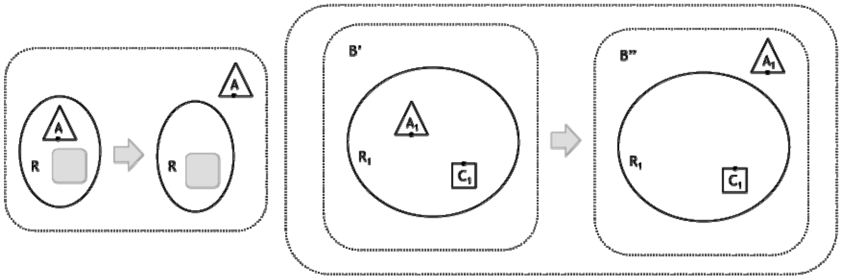

Fig. 5. OUT_OF rule applied to bigraph B′

including a placeholder (illustrated with an empty grey region) for things in a room whose placement doesn't change when the agent leaves the room. This rule and the result of applying it to bigraph B′ to produce bigraph B″ are shown in Figure 5.

To apply the rule we replace the placeholder region with the actual contents of the room besides the agent (e.g., in Figure 5 the computer is also in the room). The converses of these rules (LINKTO and INTO) are defined in the obvious ways by reversing the arrows.

3.2 Dynamic Schemas in Bigraphs

In order to represent scenes in which an agent encounters a locked room physical FORCE schemas are required. For REMOVAL_OF_RESTRAINT Johnson (1987) suggested that a force can move forward when either a barrier is not there or when it is removed by another force. Figure 6 illustrates the simplest example of this schema, an INTO rule (the converse of OUT_OF), in which an agent can enter a region that has no barrier, or had a barrier that was previously removed by another force.

If a region is locked (barricaded) then a BLOCKAGE schema must be invoked first. In the following examples agents are the forces seeking to move forward, "locks" are barriers, and "keys" are the forces that remove or bypass barriers. Given this choice, we must determine the relations between agents and keys, and between the locks and regions. Figure 7 shows three possibilities.

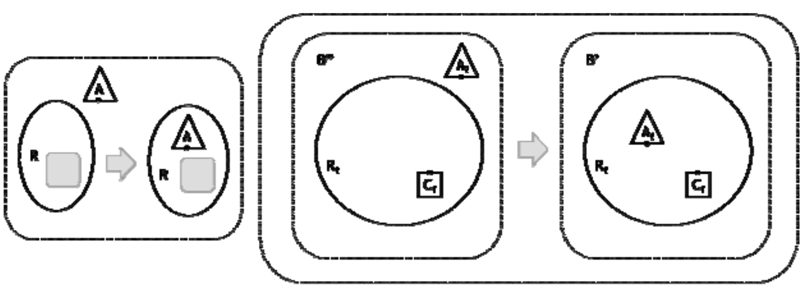

Fig. 6. Simplest REMOVAL_OF_RESTRAINT

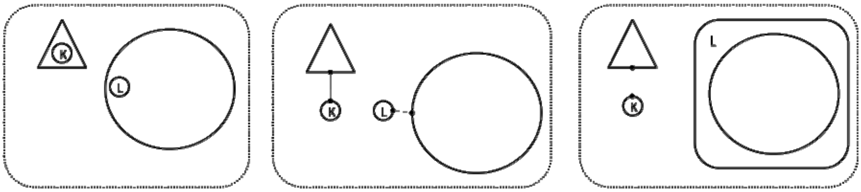

Fig. 7. Three Agent-Key, Lock-Region relations

In the first scene the agent possesses (contains) the key and the lock is in the region. In the second scene the agent and the region are only linked to the key and lock, and in the third the agent is linked to a key but the lock is represented as a barrier surrounding the region. The choice of relationships is dependent on the scene that is to be modeled. According to Johnson BLOCKAGE includes at least three possible reactions of a force when it encounters a barrier – it could go through the barrier (or remove it), go around (bypass) it, or take off in another direction.

Figure 8 illustrates one possible formalization of BLOCKAGE followed by REMOVAL_OF_RESTRAINT. The key is in (possessed by) the agent and the lock is in the room. The key links to the lock (encounters the barrier) and bypasses it. The agent is free to enter the room once the lock is bypassed, and the lock link is released, permitting other keys to engage the lock in future reactions. As before, we include a grey placeholder region in the rules to indicate that other entities can be in the room when the agent enters it. The placeholder would be filled in with the actual contents of the room when the rule was used to modify a bigraph.

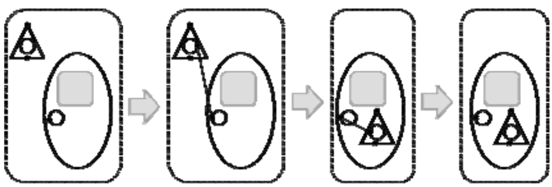

Fig. 8. BLOCKAGE followed by REMOVAL_OF_RESTRAINT

Figure 9 presents an alternative version in which the key is linked to the agent but the lock is a barricade around the region. The key links to the lock (encounters the barrier) and removes it. Being unneeded, it also disappears (this could correspond, for example, to an agent using a pass to gain entry to an area and then discarding it). The agent is free to enter the region once the lock (barrier) has been removed. Again, the empty grey inner region indicates that the rules could be applied to settings in which there are other entities in the region that the agent enters.

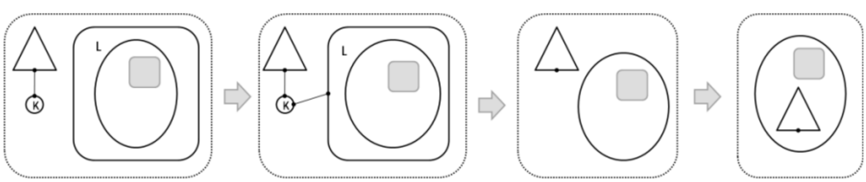

Fig. 9. BLOCKAGE followed by REMOVAL_OF_RESTRAINT (alternate)

The first step in both examples is an instance of a LINKTO reaction rule, and a later step is an instance of an INTO reaction rule. You would not want to have a more general INTO rule in the set of atomic rules allowing an agent to enter a locked room directly, or a rule that permitted the key to link to a lock it was not intended for. The property of a key being the right key to open a lock is not typically shown explicitly in a bigraph visualization but would be part of the algebraic specification. These are only two possible realizations of the schemas. The manner in which particular reaction rules and schemas are defined is dependent on the properties, relationships, and behaviors of the entities in the particular domain being modeled.

Bigraphs can be modified by the application of reaction rules, which typically change a single place or link relationship. Another way to create a new bigraph is by composition with other bigraphs to provide additional contextual information.

4 Modifying Granularity though Bigraph Composition

Bigraph composition typically does not modify existing relationships in a bigraph in the way that reaction rules do, but it may expand or refine them. For example, if there is an open link on a conference call (indicating that additional agents could participate) then composition could add a participant to the call but would not remove any of the current participants. This is similar to the case shown previously when a reaction rule with a room containing a placeholder region was used to modify a specific bigraph by filling in the open place with the actual contents of the room when applying the rule. Enhancing bigraphs in this manner allows us to define settings for other bigraphs that permit decreasing and increasing the level of scene granularity.

4.1 Bigraph Interfaces

Composition between bigraphs is performed by merging *interfaces*. An interface is a minimal specification of the portions of a particular bigraph that support additional *placings* or *linkings* (openings for more containment or linking information). If there are no such openings (e.g., the bigraphs in Figures 1-3) then the bigraph has the trivial empty interface.

A *place* in a bigraph is a *node* or a *root* (outer region) or *site* (inner region). Links can also be expanded with *inner names* and *outer names* for open links that support additional connectivity. Bigraph H in Figure 10 shows a room R_1 containing a computer C_1 and a site (0), a placeholder for other entities that can appear in rooms. The computer also has an open link x_1 for connecting to other nodes.

Bigraphs can only be composed if their interfaces match. When bigraphs H and I are composed their interface (open link x_1 and place 0) merge and disappear, resulting in bigraph J, where the agent appears in the room linked to the computer. In the merged place and link graphs J^P and J^L all the open places and links have disappeared.

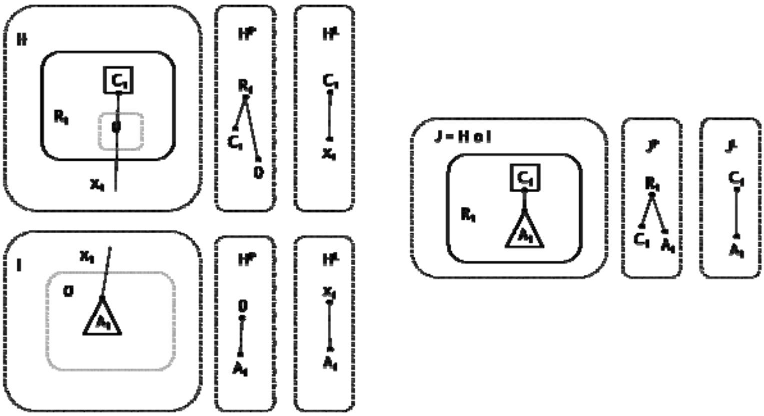

Fig. 10. Bigraph Composition

5 Built Environment Example

Consider a situation in which an agent with a key is confronted by a barrier (lock). Some information is missing, including whether she has the right key or not, whether she can reach the barrier and what is actually inside the barrier. Figure 11 shows one possible representation of this scene, a bigraph with open links and places.

Bigraph I contains a scene in which there are two unspecified outer places (roots 1 and 2). These might not be actual regions; the locality of the agent with respect to the barrier is currently unknown, but possibly different. In the first open place an agent is linked to a key for an unknown lock (indicated by open link x_1). In the second a lock (barrier) has an unknown key (open link x_2) and guards an unknown place (3). Suppose that we also have a coarser-grained view of the scene, a host bigraph H containing only an edge with two connected open links (x_1 and x_2) and open places (sites 1 and 2). Figure 12 shows bigraphs H, I and also J, which describes a scene with a single unspecified place (3) containing a room.

The bigraphs can be composed in any order. Composing H and I adds additional information to the scene. Open links with the same name merge, indicating that the key is linked to the lock. The open outer places 1 and 2 in bigraph I merge with their open place counterparts in bigraph H and disappear, which means that the agent, key, and lock are in the same place. Together, these merges mean that the agent has the right key for the lock, and that she can reach it since she is in the same place it is. Composing I with J adds the information that the entity guarded by the lock is the

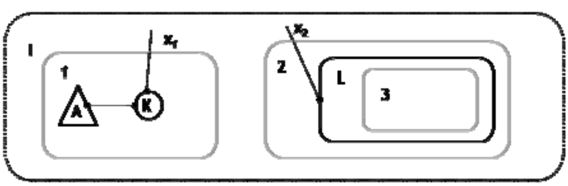

Fig. 11. Bigraph I

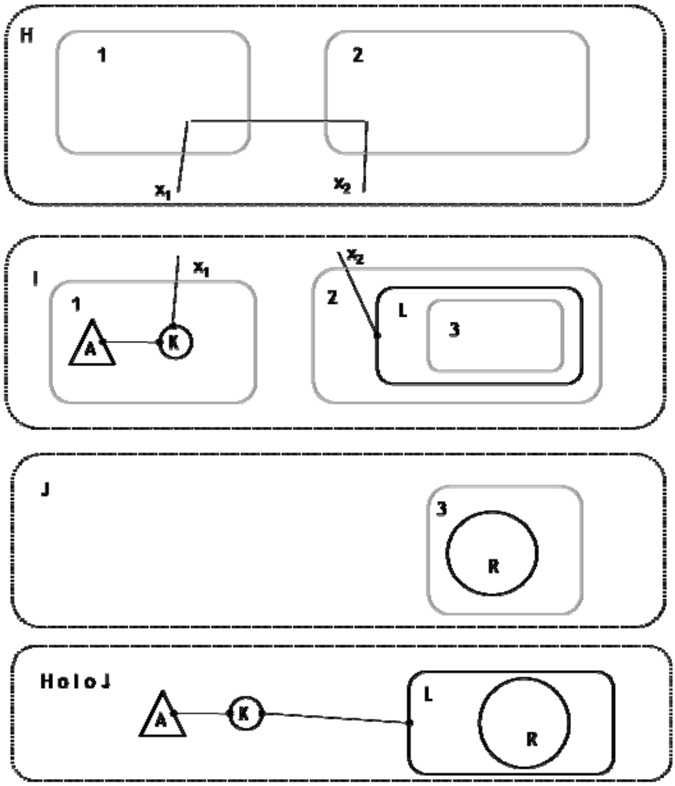

Fig. 12. Bigraphs H, I, J and their composition

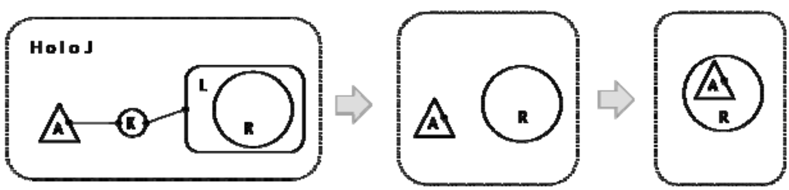

Fig. 13. Invoking BLOCKAGE and REMOVAL_OF_RESTRAINT

room R. The result of the double composition is the bottom bigraph, which provides the finest grained view of the scene.

Now that we have a more complete representation of the scene we can invoke images schemas BLOCKAGE and REMOVAL_OF_RESTRAINT through the application of reaction rules to model removing the barrier and allowing the agent to enter the room. Figure 13 shows an application of the rules from Figure 9.

In the original view of the scene shown in bigraph I, it would have been difficult to model these actions with reaction rules because it was not known which lock (barrier) the key was for, whether the agent was in the same place as the lock, and what exactly the lock was guarding. Bigraphs with open links and places support the description of scenes with incomplete information, and composing bigraphs can increase the granularity of scenes by providing additional contextual information.

6 Conclusions and Future Work

This paper has provided a method for formally representing and visualizing topological and physical image schemas for built environments. Our primary goal was to demonstrate that bigraphical models were appropriate formalization and visualization tools for representing static and dynamic image schemas. Our examples take a subset of existing topological and physical image schemas and describe them using bigraph models in the context of built environments. Static schemas CONTAINER and LINK are represented using individual bigraph subcomponents with natural visual representations. Dynamic schemas (such as INTO and LINKTO) are captured as bigraph reaction rules and visualized as a progression from one static bigraph component to another. More complex dynamic schemas, such as BLOCKAGE and REMOVAL_OF_RESTRAINT, were derived using sequences of bigraph reaction rules for static schemas. These have many possible formalizations using reaction rules, and we demonstrated a few using variants of INTO, LINKTO, and their converses. This suggests the existence of classes of reaction rules for certain schemas, and the possibility that more general specifications involving type classes of entities could be leveraged to form more general theories. We also demonstrated that bigraphs could be used to represent scenes with missing information, and that adjusting the granularity of scenes was possible using bigraph composition to add extra context.

Ontologies for built spaces are of interest, and we are developing one for hybrid outdoor-indoor spaces for built environments based on a typology of the space and the entities it contains. Types for reactive systems are ongoing areas of research (Birkedal, Debois, and Hildebrandt 2006, Bundgaard and Sassone 2006), and could be used to guide an ontological categorization of entities, relations, and behaviors in outdoor-indoor built environments, as Kuhn (2007) did in his ontological categorization of image schemas and affordances (Gibson 1977).

An algebraic theory for bigraphs for built spaces is being developed and realized as algebraic specifications in the functional language Haskell, which provides rich support for defining and reasoning about algebraic concepts such as monoids and their accompanying laws. Milner's bigraphs form a strict symmetric monoidal category (Milner 2009) with the bigraphs serving as the arrows and the interfaces serving as the objects in the category. Given this characterization, bigraph composition (used, for example, to modify scene granularity) is well-defined and avoids some of the issues that arise in the composition of arbitrary graphs.

The possible behaviors of entities in certain specific environmental settings are also of interest, and we believe that modeling affordances in bigraphs would improve behavioral specifications. In the original ambient calculus reaction rules were established based on the abilities of certain ambients (processes) to perform actions in the context they are in. Similar reasoning should be possible in a hybrid virtual-physical environment for the richer classes of ambients used in bigraphs.

Regarding the bigraph formalism itself, future work needs to focus on supporting a larger set of spatial relations (e.g., adjacency and overlap) and the kinds of spatial reasoning that are customarily available in GIS. While it is possible to describe spatial relations using links, it would be desirable to have a formal representation more closely tied to the simple spatial relation visualizations we have become accustomed to. Worboys is currently developing formal spatial enhancements to bigraphs that would support richer spatial reasoning and inference tools.

We would also like to develop support for reasoning about image schematic inference patterns in the bigraph algebra. Since inference patterns have been shown to be different in large and small scale environments (Egenhofer and Rodriguez 1999), once we have established the appropriate pattern formalisms we intend to examine whether or not these differences are still a major factor in our outdoor and indoor built environments.

Another possible area of improvement is adding support for directed movement. We may develop directed bigraphs built on top of Worboys' richer spatial bigraph model, or take advantage of existing directed bigraph extensions (Grohmann and Miculan 2007). An alternative or complementary approach would be to follow Kuhn's strategy (Kuhn 2007) and create PATH from LINK and SUPPORT schemas. We could generate series of bigraph snapshot sequences produced by the application of appropriate LINK and SUPPORT reaction rules. It remains to be seen how best to incorporate direction or more quantitative measures of location and distance should they be necessary.

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