

## Research Article

### *Event-based topology for dynamic planar areal objects*

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Representation and reasoning about dynamic spatial phenomena requires at its foundation a formalism of spatial change. This paper extends our understanding of topological change, by providing a classification and analysis of events associated with changes in topological structures of spatial areal objects as they evolve through time. Tree structures are employed to represent topological relationships between regions and holes of areal objects. Basic and complex changes are specified using structure-preserving mappings between trees. Further, the paper constructs a normal form, and proves that it is the ‘simplest’ form that can represent all the changes under consideration.

**Keywords:** events, topology, graphs, spatio-temporal, dynamic, field, regions

#### 1. Introduction

With the rapid development and deployment of data acquisition technology, huge amounts of data have been collected in environmental domains, such as oceanography, and meteorology (GoMOOS 2002, WDSS-II 2000). These temporally and spatially referenced data provide a low-level basis for describing the dynamic world, and are important to those who are interested in investigating and predicting the underlying events in these domains. Traditional approaches organize such data in information systems according to their spatial and temporal attributes (Güting *et al.* 2003, Abraham and Roddick 1999). Queries are constrained to retrieve data only if the time and space of interest is explicitly stated, and it is therefore difficult to extract relevant information when this information is not explicitly provided.

In the case of dynamic environmental phenomena, users are often interested in the data associated with events. It is therefore useful for information systems to support queries based on properties of events, along with their spatial and temporal attributes. This suggests that in our conceptual and data models, occurrent objects (events) should be given the same ontological status as continuant objects (things) (Worboys 2005). In other words, we should first abstract the data into the higher-level data type, *event*, then represent events according to various event properties and relationships, and finally support query manipulation and integration of the raw data in terms of events.

To enable such an event-oriented approach, one of the crucial tasks is to specify and distinguish different kinds of spatial events. Changes in the topological structure of regions, such as changes regarding holes, are an important aspect of spatial

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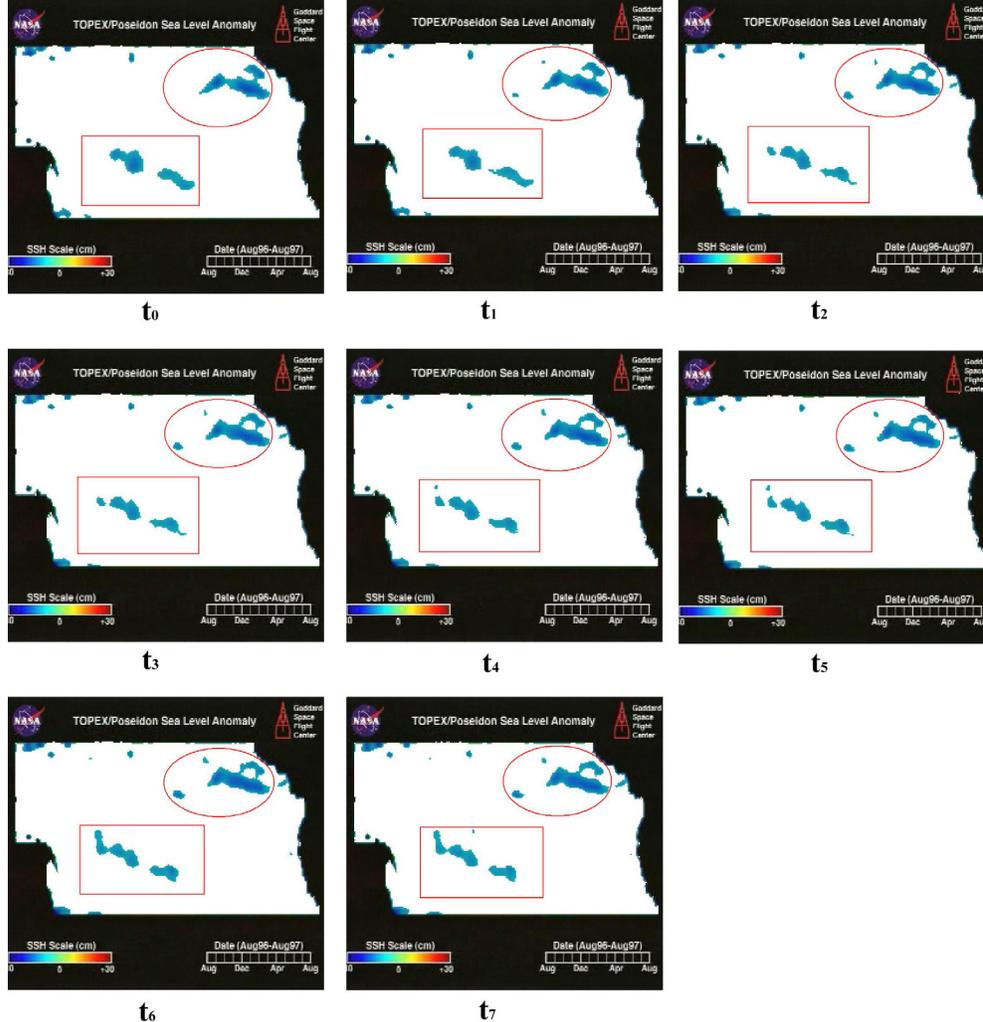


Figure 1. Consecutive snapshots of sea surface height

events. We call events specified according to the changes in topological structure *topological events*.

Consider the following examples of topological events from the domain of meteorology. Figure 1 shows eight consecutive snapshots of ocean areas with sea surface height (SSH) below a threshold (-15 cm) at the beginning of the onslaught of El Niño (Shirah 1997). It is easy to identify several changes in topological structure during this period, including:

- (1) Within the area indicated by an ellipse, between  $t_0$  and  $t_1$ , two regions *appear*. Between  $t_1$  and  $t_2$ , a region *merges* with itself and forms a *hole*, and between  $t_2$  and  $t_7$ , the *hole* is *merged* back to the exterior.
- (2) Within the area indicated by a rectangle, between  $t_0$  and  $t_2$ , a new region *appears*. Between  $t_2$  and  $t_4$ , another new region *appears*. Between  $t_4$  and  $t_6$ , three regions *merge* together. Finally, between  $t_6$  and  $t_7$ , a third region *appears*.

This paper focuses on classification and analysis of different kinds of topological events. Our approach is to take each temporal snapshot (Galton 2004) of the phenomenon, and abstract from it the structure of areal objects associated with the event at that time. This structure is formally modeled as a tree. In this way, spatial events are represented by changes in the region structure, modeled by changes

(morphisms) of the tree. The main result of our work is that each complex topological event can be expressed as a composition of basic changes, structured as a normal form.

The paper is organized as follows. In section 2 we discuss work related to the event-based approach for modeling dynamic phenomena. In section 3, we define basic and complex changes, represent them by means of trees and tree morphisms, and use these to analyze the equivalent topological changes. Section 4 presents a normal form for the complex changes, and proves that every change is equivalent to a change expressed using the normal form. Section 5 shows the properties that a normal form must have. Finally, in section 6 we draw conclusions and discuss future research directions.

## 2. Background

In this section we provide a background to research on modeling dynamic phenomena. Abraham and Roddick (1999) give a good survey of early work on spatio-temporal information systems. In most approaches, the occurments (events, changes, and other dynamic happenings) are represented implicitly. More recently, there is an increasing interest in representing occurments explicitly, and upgrading them to an equal status with continuant objects. Grenon and Smith (2004) in their SPAN ontology define occurments to be entities which unfold themselves through time in their successive temporal parts. Worboys and Hornsby (2004) emphasize the similarities between events and objects in data models, and propose a unified modeling approach for both events and objects. Worboys (2005) presents a pure event oriented model, in which real-world happenings are represented using algebraic approaches. Galton (2004) points out that there is not a clean separation between (spatial) objects and (temporal) events, since in the real world we can find many phenomena (e.g., storms, floods), which are not easy to fit into a simple object/event dichotomy.

Some research has focused on exploring and categorizing different kinds of events or changes. Based on three basic components of geo-objects identified by Armstrong (1988), eight change scenarios are proposed by Roshannejad and Kainz (1995), including change in geometry, change in topology, change in attribute, change in topology and attribute, change in attribute and geometry, change in geometry and topology, change in geometry, topology and attribute, and no change. Hornsby and Egenhofer (2000) present research on specifying identification-based change. They systematically analyze change with respect to states of existence and nonexistence for identifiable objects. Claramunt (1995) analyzes possible changes that can take place in entities and categorizes them into three different classes: evolution concerning a single entity, evolution in the functional relations between several entities, and evolution in spatial structure involving several entities. Renolen (2000) defines seven types of change: creation, alteration, destruction, reincarnation, split, merge, and reallocation.

Besides defining and classifying change in general, some work has focused on event-based modeling in specific domains. Galton and Worboys (2005) discuss the ontological categories of events and states in dynamic geo-networks (accidents, flows, etc.), and possible kinds of causal relations between events. Cole and Hornsby (2005) identify and model some significant occurments in a harbor as noteworthy events.

Another research direction related to topological change is the analysis of transition between binary topological relations. Examples of this work include (Randell *et al.* 1992, Egenhofer and Al-Taha 1992). Research in this direction aims at build-

ing conceptual neighborhood graphs (or transition graphs) in order to describe all direct transitions between certain kinds of binary topological relation during the continuous change of two spatial objects. This research has similarities to our work; however, study of transitions between topological relations is different from the study of changes in topological structure. In the former, the emphasis is on exploring the constraints that continuity imposes on transitions between binary topological relationships. In our work, the topological structure of spatial objects is more complex, for example allowing objects to have holes, and our analysis focuses on changes in the structure of such complex objects.

### 3. Basic and complex topological changes

This paper discusses the topological characterization of changes to areal objects as they evolve through time. We assume that the areal objects are embedded in a 2-dimensional spatial domain, and the evolution is continuous (no sudden jumps). We do not assume that the areal object is simply connected (so it may have holes), or even connected (so it may have several disconnected components). A rooted tree model is presented to model the topological structure of areal objects at a snapshot in time. As the areal object evolves through time, the corresponding tree will change. We will use changes to the tree to represent changes to the areal object. In this section, we first show that the components of an areal object form a hierarchy that can be represented by a tree. Then we review formal definitions of trees and tree morphisms, and lastly use them to specify different kinds of changes.

#### 3.1. Areal object and tree representation

**Definition 1** An *areal object* is a set of points  $R$  in 2-dimensional space with the following properties:

- (1)  $R$  is regular closed, that is the closure of the interior of  $R$  is  $R$  (Schneider and Behr 2006).
- (2)  $R$  is bounded, that is  $R$  is contained in the ‘inside’ of a Jordan curve (Fulton 1995) in the spatial domain.
- (3) Both  $R$  and its complement have a finite number of connected components.

Given an areal object  $R$ , connected components of  $R$  are said to be its *positive components* and connected components of the complement of  $R$  are said to be its *negative components*. Both positive and negative components are referred to as *components*. In this work, we are not concerned with the boundaries of components. We note that components of an areal object form a partition of the whole space. An areal object must have one and may have more than one positive component as well as more than one negative component, and both positive and negative components may have holes. As an example, Figure 2(a) shows an areal object, whose positive and negative components are represented by the shaded areas in Figure 2(b) and 2(c), respectively. In the figure, the outer rectangles represent the extent of the spatial domain.

We are concerned with the ‘surrounded by’ relation between components. For example, in Figure 2(a), components 3 and 4 are surrounded by component 2. Components 2,3 and 4 are surrounded by component 1. Components 1,2,3 and 4 are surrounded by component 0. The ‘surrounded by’ relation satisfies the following properties, mentioned in (Bittner and Donnelly 2007):

- (1) Transitivity: for any components  $C_1$ ,  $C_2$ , and  $C_3$ , whenever  $C_1$  is surrounded

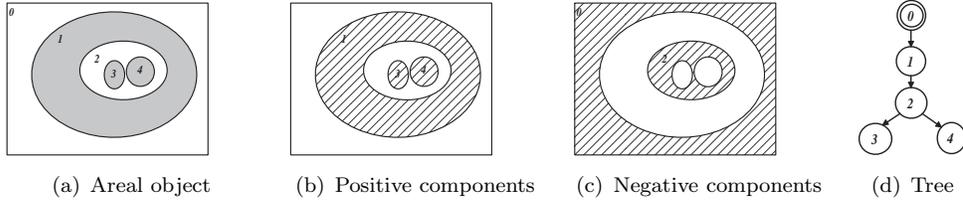


Figure 2. An example of tree representation of components

- by  $C_2$ , and  $C_2$  is surrounded by  $C_3$ , we have  $C_1$  is surrounded by  $C_3$ .
- (2) Asymmetry: for any components  $C_1$  and  $C_2$ , if  $C_1$  is surrounded by  $C_2$ , then  $C_2$  cannot be surrounded by  $C_1$ .
  - (3) The root property: there is one and only one component  $C$  such that all the other components are surrounded by  $C$ .
  - (4) No-partial-overlap: for any different components  $C_1$  and  $C_2$  surrounding the same component, it holds that either  $C_1$  is surrounded by  $C_2$ , or  $C_2$  is surrounded by  $C_1$ .

According to these properties, rooted tree structures are employed to represent the ‘surrounded by’ relation between all the components (Worboys and Bofakos 1993). A rooted tree is a special type of directed tree, in which edges have a natural orientation (being away from the root). In the rooted tree, each vertex represents a component, and the direct descendants of each vertex represent the components that are immediately surrounded by the component it represents. A component  $C_1$  is surrounded by a component  $C_2$  if and only if there is a directed path from the vertex representing  $C_2$  to the vertex representing  $C_1$ . Figure 2(d) shows such a tree representation of the areal object in Figure 2(a). The root of the tree is doubly circled and directions of edges are indicated by arrows.

### 3.2. Basic definitions

The following gives the formal definitions that relate to the notations of tree and tree morphism. Note: in this and the following sections, trees are always rooted trees.

**Definition 2** A *graph*  $G$  is a pair  $\langle V, E \rangle$ , where  $V$  is the set of vertices in the graph, and  $E$  is a set of subsets of  $V$  representing the edges of the graph. Each element in  $E$  has the form  $\{v_1, v_2\}$ , where  $v_1$  and  $v_2$  are different vertices in  $V$ .

**Definition 3** In a graph  $G = \langle V, E \rangle$ , a *path* is a sequence of vertices of the form:

$$[v_1, v_2, \dots, v_{k-1}, v_k]$$

where  $v_i \in V$  for  $i = 1, 2, \dots, k$ , and  $\{v_i, v_{i+1}\} \in E$  for  $i = 1, 2, \dots, k - 1$ .

A path is defined to be *simple*, if  $\forall i, j \in \{1, 2, \dots, k\}$ ,  $i \neq j$  implies  $v_i \neq v_j$ .

A *cycle* is defined to be a path  $[v_1, v_2, \dots, v_{k-1}, v_k]$  such that  $v_1 = v_k$ . A cycle is defined to be *simple*, if  $\forall i, j \in \{1, 2, \dots, k - 1\}$ ,  $i \neq j$  implies  $v_i \neq v_j$ .

We will use  $V(G)$ ,  $E(G)$  to represent the set of vertices and edges of the graph  $G$  respectively. We say vertices  $v_1$  and  $v_2$  are *adjacent* in  $G$ , if  $\{v_1, v_2\} \in E(G)$ .

**Definition 4** A *graph morphism* from a graph  $G_1$  to a graph  $G_2$  is a function  $\varphi : V(G_1) \rightarrow V(G_2)$  such that for all  $v_i, v_j \in V(G_1)$ ,  $\varphi(v_i)$  and  $\varphi(v_j)$  are adjacent in  $G_2$  whenever  $v_i$  and  $v_j$  are adjacent in  $G_1$ .

**Definition 5** A *tree*  $T$  is defined to be a graph with the properties that:

- (1) For all different vertices  $v_1, v_2$ , there is one and only one simple path in  $T$  that connects  $v_1$  and  $v_2$ .
- (2) There is a distinguished vertex  $r$  called the *root* of the tree.

To emphasize the root of the tree, we will use a triple  $\langle V, E, r \rangle$  to represent a tree.

**Definition 6** Given two trees  $T_1 = \langle V_1, E_1, r_1 \rangle$  and  $T_2 = \langle V_2, E_2, r_2 \rangle$ , a *tree morphism*  $\varphi$  from  $T_1$  to  $T_2$  is defined to be the graph morphism  $\varphi$  from  $T_1$  to  $T_2$ , with an additional requirement that  $\varphi(r_1) = r_2$ .

A tree morphism is *injective* if and only if  $\varphi$  is an injective function; i.e. distinct vertices of  $T_1$  are mapped to distinct vertices of  $T_2$  through  $\varphi$ . A tree morphism is *surjective* if and only if  $\varphi$  is a surjective function; i.e. every vertex of  $T_2$  is mapped onto by at least one vertex of  $T_1$  through  $\varphi$ . A tree morphism is an *isomorphism*, if and only if it is both an injective and a surjective tree morphism.

### 3.3. Basic topological changes

Topological changes occur when components appear, disappear, merge, split, etc. All of these topological changes have correspondences to changes in the tree structure. Our first step is to define some basic changes of the tree structure, from which all the changes of interest can be constructed.

Suppose  $T_1$  and  $T_2$  are trees representing the topological structures of an areal object at time  $t_1$  and  $t_2$  ( $t_1 < t_2$ ). A basic change is denoted by the expression ' $T_1 \xrightarrow{\gamma} T_2$ ', where  $\gamma$  is the change, and can be represented by a single morphism  $\varphi$ , either from  $T_1$  to  $T_2$  or from  $T_2$  to  $T_1$ . Five types of basic change can be specified according to the properties of the tree morphisms between  $T_1$  and  $T_2$ .

**Definition 7** A basic change  $T_1 \xrightarrow{\gamma} T_2$  is one of the following types:

- (1)  $T_1 \xrightarrow{\gamma} T_2$  is of type **insert**, if the effect of the change can be represented by an injective but not surjective tree morphism  $\varphi$  from  $T_1$  to  $T_2$ .
- (2)  $T_1 \xrightarrow{\gamma} T_2$  is of type **merge**, if the effect of the change can be represented by a surjective but not injective tree morphism  $\varphi$  from  $T_1$  to  $T_2$ .
- (3)  $T_1 \xrightarrow{\gamma} T_2$  is of type **delete**, if the effect of the change can be represented by an injective but not surjective tree morphism  $\varphi$  from  $T_2$  to  $T_1$ .
- (4)  $T_1 \xrightarrow{\gamma} T_2$  is of type **split**, if the effect of the change can be represented by a surjective but not injective tree morphism  $\varphi$  from  $T_2$  to  $T_1$ .
- (5)  $T_1 \xrightarrow{\gamma} T_2$  is of type **no change**, if the effect of the change can be represented by a tree isomorphism  $\varphi$  from  $T_1$  to  $T_2$ .

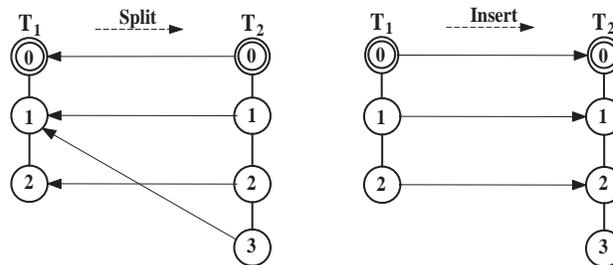


Figure 3. Representation of different topological changes

As an example, Figure 3 shows representations of a basic split and a basic insert

between times  $t_1$  and  $t_2$ . The dashed arrows show the direction of changes, and solid arrows between vertices show the morphisms. Both representations indicate different ways that the topological structure of an areal object changes. A change modeled by the basic split is shown in Figure 4(A), in which component 2 evolves its shape to engulf a new component 3. A change modeled by the basic insert is shown in Figure 4(B), in which component 3 arises differently, this time emerging and growing inside component 2.

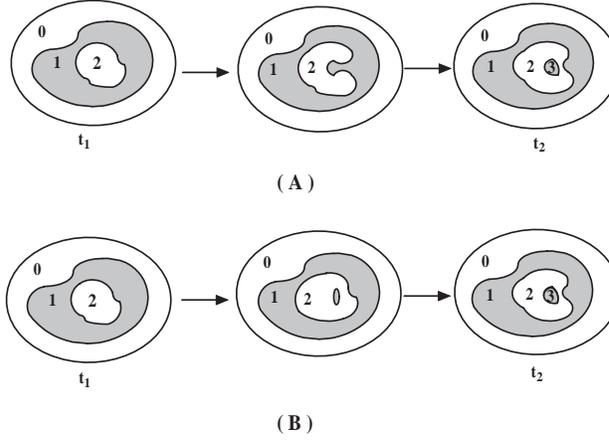


Figure 4. Two different topological changes

### 3.4. Complex topological changes

As an areal object evolves through time, a sequence of basic changes is established. We define a sequence of basic changes to be a complex change. The definition is as follows:

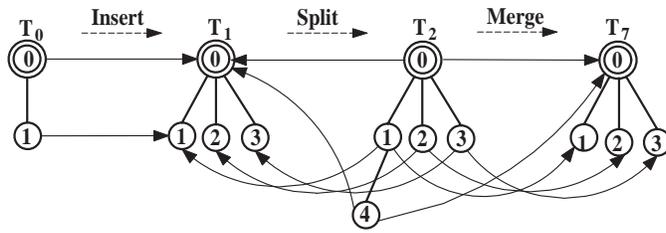
**Definition 8** A *complex change* from  $T_0$  to  $T_n$  is of the form

$$T_0 \xrightarrow{\gamma_0} T_1 \xrightarrow{\gamma_1} T_2 \xrightarrow{\gamma_2} \dots T_i \xrightarrow{\gamma_i} \dots \xrightarrow{\gamma_{n-2}} T_{n-1} \xrightarrow{\gamma_{n-1}} T_n$$

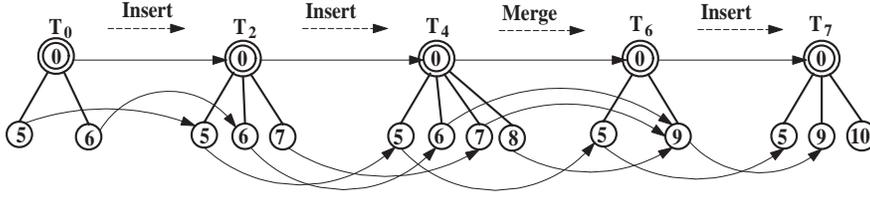
in which each  $T_i \xrightarrow{\gamma_i} T_{i+1}$  ( $0 \leq i \leq n-1$ ) represents a basic change from  $T_i$  to  $T_{i+1}$ .

For simplicity, in representations we omit the *no change* events within a complex change. If there is a *no change* event from  $T_i$  to  $T_{i+1}$ , both  $T_i$  and  $T_{i+1}$  have the same structure. Therefore the basic change starting from  $T_{i+1}$  can be replaced by a change that starts from  $T_i$ . In this way, the *no change* event together with the tree  $T_{i+1}$  are omitted. For example, consider the case shown in Figure 1. We define the selected locations in the ellipse area and the rectangle area to be areal objects  $R_1$  and  $R_2$  respectively. The evolution of  $R_1$  and  $R_2$  can be represented by complex changes  $C_1$  and  $C_2$  as shown in Figures 5(a) and 5(b) respectively. In complex change  $C_1$  we omit the *no change* events starting from  $T_2$ ,  $T_3$ ,  $T_4$ , and  $T_5$ . In complex change  $C_2$  we omit the *no change* events starting from  $T_0$ ,  $T_2$ , and  $T_4$ .

A basic issue is to define the notion of equivalence of complex changes. For example, Figure 6 shows two equivalent complex changes. Both changes result in a new component denoted by vertex 5. Vertices 1 and 4 in the final state originate from vertex 1 in the original state, and vertex 6 in the final state originates from vertices 2 and 3 in the original state.

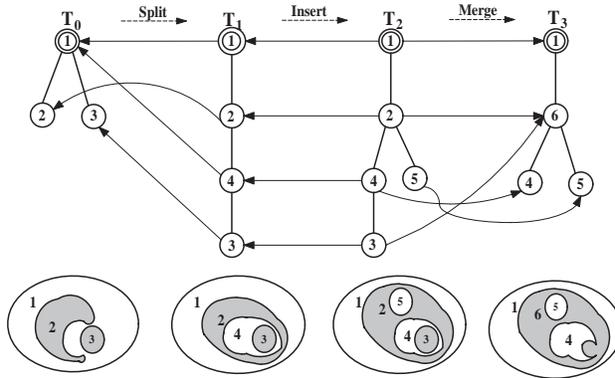


(a) Topological changes as  $R_1$  evolves

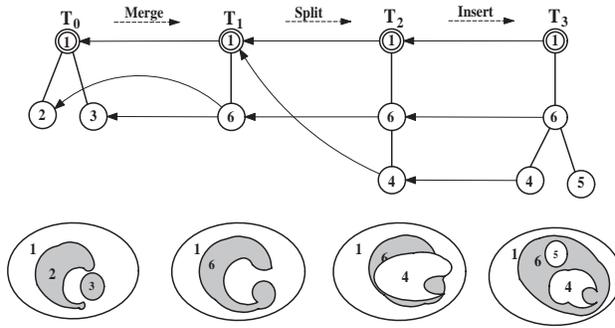


(b) topological changes as  $R_2$  evolves

Figure 5. Complex representation of the areal objects in Figure 1



(a) Complex Change A



(b) Complex Change B

Figure 6. Two equivalent complex changes

The following definitions formalize our intuitions about equivalent complex changes in terms of trees and tree morphisms.

**Definition 9** Let  $C$  be a basic change  $T_1 \xrightarrow{\gamma} T_2$  specified by a morphism  $\varphi$ .  $C$  induces a *transform-to relation*  $R \subset V(T_1) \times V(T_2)$  such that  $\forall v_1 \in V(T_1), v_2 \in V(T_2), (v_1, v_2) \in R$  if and only if either  $\varphi(v_1) = v_2$  or  $\varphi(v_2) = v_1$ .

**Definition 10** Let  $C$  be a complex change from  $T_0$  to  $T_n$ :

$$T_0 \xrightarrow{\gamma_0} T_1 \xrightarrow{\gamma_1} T_2 \xrightarrow{\gamma_2} \dots T_i \xrightarrow{\gamma_i} \dots \xrightarrow{\gamma_{n-2}} T_{n-1} \xrightarrow{\gamma_{n-1}} T_n,$$

and let  $R_k(0 \leq k \leq n-1)$  denote the transform-to relation induced by the basic change from  $T_k$  to  $T_{k+1}$ .

- (1) For any  $0 \leq i \leq n-1$ , the *future* of  $v \in V(T_i)$  from stage  $i$  is defined to be  $F(v, i) = \{w \in V(T_n) | (v, w) \in R_i \circ R_{i+1} \circ \dots \circ R_{n-1}\}$ . The *future* of  $v \in V(T_n)$  from stage  $n$  is defined to be  $F(v, n) = \{v\}$ .
- (2) For any  $1 \leq i \leq n$ , the *past* of  $v \in V(T_i)$  from stage  $i$  is defined to be  $P(v, i) = \{w \in V(T_0) | (w, v) \in R_0 \circ R_1 \circ \dots \circ R_{i-1}\}$ . The *past* of  $v \in V(T_0)$  from stage 0 is defined to be  $P(v, 0) = \{v\}$ .
- (3) The *set of essential insertions* of  $C$  is defined to be  $I(C) = \{(v, i) | i \in \{1, 2, \dots, n\} \wedge v \in V(T_i) \wedge F(v, i) \neq \emptyset \wedge P(v, i) = \emptyset\}$ .

In definition 10, the future of a vertex  $v$  is the set of vertices in the final state to which  $v$  transforms. The past of  $v$  is the set of vertices in the initial state which transform to  $v$ . An essential insertion  $(v, i)$  refers to a vertex  $v$  at stage  $i$  that is introduced by a basic insert, and which transforms to some vertices in the final state.

Two changes are defined to be equivalent if and only if both changes start from the same tree  $T_0$ , end at the same tree  $T_n$ , have the same set of essential insertions  $I$ , and have the same transform-to relation, characterized by the future functions, from the vertices of  $T_0$  and  $I$  to the vertices of  $T_n$ . The formal definition is given as follows:

**Definition 11** Let  $C$  and  $C'$  be two changes from  $T_0$  to  $T_n$ , and from  $T'_0$  to  $T'_n$ , respectively. Let  $F$  and  $F'$  be the future functions of  $C$  and  $C'$ . Let  $I(C)$  and  $I(C')$  be the essential insertion sets of  $C$  and  $C'$ .

$C$  is defined to be *equivalent* to  $C'$  if and only if there is a tree isomorphism  $\varphi_0$  from  $V(T_0)$  to  $V(T'_0)$ , a tree isomorphism  $\varphi_n$  from  $V(T_n)$  to  $V(T'_n)$ , and a bijective function  $f$  from  $I(C)$  to  $I(C')$  such that,

- (1)  $\forall v \in V(T_0), \varphi_n(F(v, 0)) = F'(\varphi_0(v), 0)$ , and
- (2)  $\forall (v, i) \in I(C), \varphi_n(F(v, i)) = F'(f(v, i))$ .

#### 4. The normal form for representing complex changes

Complex change modeling some real world phenomenon can be composed of a large number of basic changes. Similar to the graph generalizations (Stell and Worboys 1999), it can be useful to represent a complex change by an equivalent change in a unified form. For example, the complex change shown in Figure 5(b) can be simplified by combining the first two basic inserts into a single basic insert. The resulting equivalent change is shown in Figure 7. Can we further simplify the resulting change? In this section, we provide a normal form and prove that any complex change can be expressed in this form.

**Definition 12** A change is defined to be in *normal form* if and only if it is composed in the given order of four basic changes, represented by:

$$T_0 \xrightarrow{\gamma_0} T_1 \xrightarrow{\gamma_1} T_2 \xrightarrow{\gamma_2} T_3 \xrightarrow{\gamma_3} T_4$$

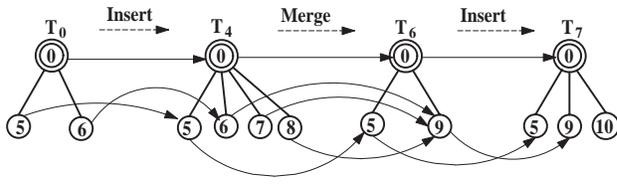


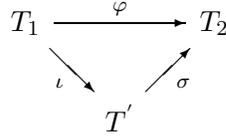
Figure 7. A simplification of complex change in Figure 5(b)

in which,  $T_0 \xrightarrow{\gamma_0} T_1$  is a basic insert (or no change).  $T_1 \xrightarrow{\gamma_1} T_2$  is a basic split (or no change).  $T_2 \xrightarrow{\gamma_2} T_3$  is a basic merge (or no change).  $T_3 \xrightarrow{\gamma_3} T_4$  is a basic delete (or no change).

In this section, we prove that any complex change is equivalent to a change represented in normal form. We begin by presenting some technical results on trees and morphisms used to prove our main results. The proofs and detailed interpretations of the three lemmas are presented in appendix A. Note: we use  $\varphi$  to represent a general function,  $\iota$  to represent an injective function, and  $\sigma$  to represent a surjective function. We also use  $I$ ,  $S$ ,  $M$ ,  $D$  as abbreviations of insert (or no change), basic split (or no change), basic merge (or no change), and basic delete (or no change) respectively.

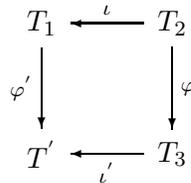
**Lemma 1** Let  $T_1$  and  $T_2$  be trees, and  $\varphi$  be a tree morphism from  $T_1$  to  $T_2$ . Then, it is possible to find another tree  $T'$ , an injective tree morphism  $\iota$  from  $T_1$  to  $T'$ , and a surjective tree morphism  $\sigma$  from  $T'$  to  $T_2$ , satisfying:

- (1)  $\iota \circ \sigma = \varphi$ .
- (2) Let  $S_1 = V(T') \setminus \text{img}(\iota)$  and  $S_2 = V(T_2) \setminus \text{img}(\varphi)$ . Then  $\sigma$  defines a bijection between  $S_1$  and  $S_2$ , by restricting the domain of  $\sigma$  to  $S_1$ .



**Lemma 2** Let  $T_2$  and  $T_3$  be trees,  $\iota$  be an injective tree morphism from  $T_2$  to  $T_1$ , and  $\varphi$  be a tree morphism from  $T_2$  to  $T_3$ . Then, it is possible to find a tree  $T'$ , an injective tree morphism  $\iota'$  from  $T_3$  to  $T'$ , and a tree morphism  $\varphi'$  from  $T_1$  to  $T'$ , satisfying:

- (1)  $\iota^{-1} \circ \varphi = \varphi' \circ \iota'^{-1}$ .
- (2) If  $\varphi$  is surjective, then  $\varphi'$  is surjective. If  $\varphi$  is injective, then  $\varphi'$  is injective.
- (3) Let  $S_1 = V(T_1) \setminus \text{img}(\iota)$  and  $S_2 = V(T') \setminus \text{img}(\iota')$ . Then  $\varphi'$  defines a bijection between  $S_1$  and  $S_2$ , by restricting the domain of  $\varphi'$  to  $S_1$ .
- (4) Let  $S_3 = V(T_3) \setminus \text{img}(\varphi)$  and  $S_4 = V(T') \setminus \text{img}(\varphi')$ . Then  $\iota'$  defines a bijection between  $S_3$  and  $S_4$ , by restricting the domain of  $\iota'$  to  $S_3$ .



**Lemma 3** Let  $T_1, T_2$  and  $T_3$  be trees,  $\sigma_1$  be a surjective morphism from  $T_1$  to  $T_2$ , and  $\sigma_2$  be a surjective morphism from  $T_3$  to  $T_2$ . Then, it is possible to find two trees  $T'_4$  and  $T'_5$ , a surjective morphism  $\sigma'_1$  from  $T'_4$  to  $T_1$ , a surjective morphism  $\sigma'_2$  from  $T'_4$  to  $T'_5$ , and an injective morphism  $\iota'$  from  $T_3$  to  $T'_5$ , satisfying  $\sigma_1 \circ \sigma_2^{-1} = \sigma'_1 \circ \sigma'_2 \circ \iota'^{-1}$ .

$$\begin{array}{ccccc}
 T_1 & \xrightarrow{\sigma_1} & T_2 & \xleftarrow{\sigma_2} & T_3 \\
 & \searrow \sigma'_1 & & & \downarrow \iota' \\
 & & T'_4 & \xrightarrow{\sigma'_2} & T'_5
 \end{array}$$

We next discuss some special complex changes, after which the discussion will be extended to arbitrary complex changes.

**Definition 13** An *MD-change* is defined to be a complex change of the form:

$$T_0 \xrightarrow{\gamma_0} T_1 \xrightarrow{\gamma_1} T_2$$

in which,  $T_0 \xrightarrow{\gamma_0} T_1$  is a basic merge (or no change), and  $T_1 \xrightarrow{\gamma_1} T_2$  is a basic delete (or no change).

**Definition 14** An *SMD-change* is defined to be a complex change of the form:

$$T_0 \xrightarrow{\gamma_0} T_1 \xrightarrow{\gamma_1} T_2 \xrightarrow{\gamma_2} T_3$$

in which,  $T_0 \xrightarrow{\gamma_0} T_1$  is a basic split (or no change),  $T_1 \xrightarrow{\gamma_1} T_2$  is a basic merge (or no change), and  $T_2 \xrightarrow{\gamma_2} T_3$  is a basic delete (or no change).

**Theorem 1** Any complex change that is arbitrarily composed of basic merges and basic deletes is equivalent to an MD-change.

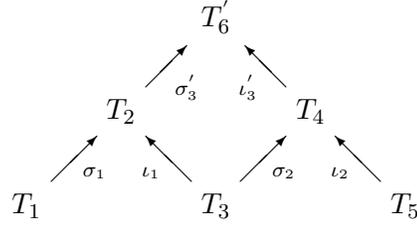
**Proof:** It is straightforward to prove this theorem if the complex change  $C$  is composed of only basic merges, or  $C$  is composed of only basic deletes. So, consider the case in which  $C$  is composed of both basic merges and basic deletes in any order. By composing adjacent basic merges into one basic merge and adjacent basic deletes into one basic delete, we are able to obtain a complex change  $C'$  which is equivalent to  $C$ , and is composed of a sequence of MD-changes.

Consider two adjacent MD-changes of the form:

$$\begin{array}{ccccccc}
 T_1 & \xrightarrow{\sigma_1} & T_2 & \xleftarrow{\iota_1} & T_3 & \xrightarrow{\sigma_2} & T_4 & \xleftarrow{\iota_2} & T_5 \\
 & & \text{MD-change} & & & & \text{MD-change} & & \\
 & & | \cdots \cdots \cdots \rightarrow & & | \cdots \cdots \cdots \rightarrow & & | & & |
 \end{array}$$

in which,  $\sigma_1, \sigma_2$  are surjective morphisms specifying basic merges, and  $\iota_1$  and  $\iota_2$  are injective tree morphisms specifying basic deletes.

Using Lemma 2, we are able to construct a tree  $T'_6$ , a surjective tree morphism  $\sigma'_3$  from  $T_2$  to  $T'_6$ , and an injective tree morphism  $\iota'_3$  from  $T_4$  to  $T'_6$ , satisfying the condition that the complex change from  $T_2$  to  $T_4$  specified by the sequence  $\sigma'_3$  and  $\iota'_3$  is equivalent to the complex change from  $T_2$  to  $T_4$  specified by the sequence  $\iota_1$  and  $\sigma_2$ . (See corollary 3 in appendix A for a detailed explanation.)



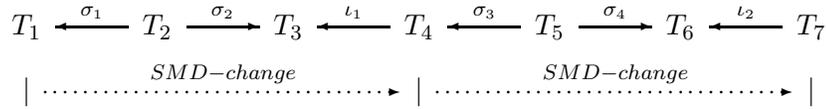
Hence, the composition of the two adjacent MD-changes will be equivalent to one MD-change, which is composed of a basic merge from  $T_1$  to  $T'_6$  specified by  $\sigma_1 \circ \sigma'_3$ , and a basic delete from  $T'_6$  to  $T_5$  specified by  $\iota_2 \circ \iota'_3$ .

Repeating this procedure, we finally get one MD-change, which is equivalent to the complex change  $C$ . □

**Theorem 2** Any complex change, which is arbitrarily composed of basic splits, basic merges, and basic deletes, is equivalent to a SMD-change.

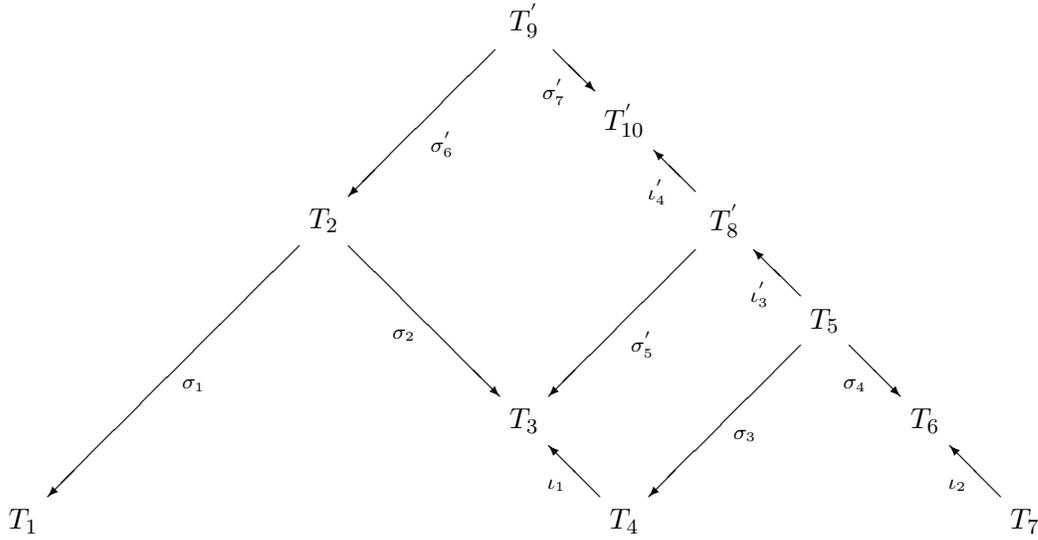
**Proof:** It is straightforward to prove this theorem if complex change  $C$  is composed of only basic splits, or is composed of basic merges and basic deletes. So, consider the case in which complex change  $C$  contains basic splits, as well as some basic merges and basic deletes in any order. By composing adjacent basic merges and basic deletes to form one MD-change and adjacent basic splits to form one basic split, we obtain a complex change  $C'$  which is equivalent to  $C$ , and is composed of a sequence of SMD-changes.

Consider two adjacent SMD-changes of the form:



in which,  $\sigma_1$  and  $\sigma_3$  are surjective morphisms specifying basic splits,  $\sigma_2$  and  $\sigma_4$  are surjective morphisms specifying basic merges, and  $\iota_1$  and  $\iota_2$  are injective morphisms specifying basic deletes. Since there is a tree morphism  $\sigma_3 \circ \iota_1$  from  $T_5$  to  $T_3$ , using Lemma 1, we are able to find a tree  $T'_8$ , an injective tree morphism  $\iota'_3$  from  $T_5$  to  $T'_8$ , and a surjective morphism  $\sigma'_5$  from  $T'_8$  to  $T_3$ , satisfying the condition that the complex change from  $T_3$  to  $T_5$  specified by the sequence  $\iota_1, \sigma_3$  is equivalent to the complex change from  $T_3$  to  $T_5$  specified by the sequence  $\sigma'_5, \iota'_3$ . (See corollary 1 in appendix A for a detailed explanation.)

Since there is a surjective tree morphism from  $T_2$  to  $T_3$  and a surjective morphism from  $T'_8$  to  $T_3$ . Then, using Lemma 3 we are able to find trees  $T'_9$  and  $T'_{10}$ , together with tree morphisms  $\sigma'_6, \sigma'_7$  and  $\iota'_4$ . Here,  $\sigma'_6$  and  $\sigma'_7$  are surjective morphisms from  $T'_9$  to  $T_2$  and from  $T'_9$  to  $T'_{10}$  respectively.  $\iota'_4$  is an injective morphism from  $T'_8$  to  $T'_{10}$ . Therefore, the complex change from  $T_2$  to  $T'_8$  specified by the sequence  $\sigma'_6, \sigma'_7$  and  $\iota'_4$  is equivalent to the complex change from  $T_2$  to  $T'_8$  specified by the sequence  $\sigma_2$  and  $\iota'_5$ .



It follows that the composition of the two adjacent SMD-changes is equivalent to one SMD-change, which is composed of a basic split specified by  $\sigma'_6 \circ \sigma_1$ , and a MD-change composed of basic merges and basic deletes specified by the sequence  $\sigma'_7, l'_4, l'_3, \sigma_4$  and  $l_2$ , respectively.

Repeating this procedure, we finally get one SMD-change, which is equivalent to the complex change  $C$ .  $\square$

We are now ready to prove the main result of this section.

**Theorem 3** Any complex change is equivalent to a complex change in normal form.

**Proof:** It is straightforward to prove this theorem if complex change  $C$  is composed of only basic inserts, or is composed of basic changes of any type except basic inserts.

So, consider the case in which  $C$  is composed of basic inserts, as well as other types of basic changes in any order. By composing adjacent basic splits, basic merges and basic deletes together to make one SMD-change, and composing adjacent basic inserts together to one basic insert, we are able to obtain a complex change  $C'$ , which is composed of a set of changes in normal form.

Using Lemmas 1 and 2, we can prove that for any complex change composed of a SMD-change followed by a basic insert, we are able to find an equivalent complex change composed of a basic insert followed by a SMD-change change. Hence, any two adjacent complex changes in normal form can be composed together to form one complex change in normal form. (See corollaries 2,4,5 in appendix A for a detailed explanation.)

Repeating this procedure, we finally get one complex change in normal form that is equivalent to the complex change  $C$ .  $\square$

## 5. Properties of the normal form

In the previous section, we introduced a normal form, and proved that every change is equivalent to a complex change in normal form. In this section we will show that

all four types of basic change are required as constituents to represent every possible change. We will also prove that no other form composed of four basic changes in a different sequence from that in normal form can represent all the changes. We first note without proof the following lemmas:

**Lemma 4** Let  $C$  be a complex change from  $T_1$  to  $T_2$ , which is composed of basic changes of any type excluding basic insert. Then, for any vertex  $v$  of  $T_2$ , there is at least one vertex of  $T_1$  that transforms to  $v$  through  $C$ .

**Lemma 5** Let  $C$  be a complex change from  $T_1$  to  $T_2$ , which is composed of basic changes of any type excluding basic split. Then, any vertex  $v$  of  $T_1$  transforms to at most one vertex of  $T_2$  through  $C$ .

**Lemma 6** Let  $C$  be a complex change from  $T_1$  to  $T_2$ , which is composed of basic changes of any type excluding basic merge. Then, for any vertex  $v$  of  $T_2$ , there is at most one vertex of  $T_1$  that transforms to  $v$  through  $C$ .

**Lemma 7** Let  $C$  be a complex change from  $T_1$  to  $T_2$ , which is composed of basic changes of any type excluding basic delete. Then, any vertex  $v$  in  $T_1$  transforms to at least one vertex of  $T_2$  through  $C$ .

Based on the four lemmas, we can prove the following theorems:

**Theorem 4** Any form, which does not require changes to be composed of all four basic changes, cannot represent all changes.

**Proof:** Consider the example shown in Figure 8. Let  $C$  be the complex change from  $T_0$  to  $T_4$ . No vertex of  $T_0$  transforms to vertex 5 of  $T_4$ . By Lemma 4, any complex change that is equivalent to  $C$  must contain a basic insert. Vertex 2 of  $T_0$  transforms to two vertices 6,7 of  $T_4$ . By Lemma 5, any change that is equivalent to  $C$  must contain a basic split. Vertices 2,3 of  $T_0$  transform to vertex 7 of  $T_4$ . By Lemma 6, any change that is equivalent to  $C$  must contain a basic merge. Vertex 4 of  $T_0$  does not transform to any vertex of  $T_4$ . By Lemma 7, any change that is equivalent to  $C$  must contain a basic delete. In all, the complex change  $C$  requires all four types of basic changes. Thus, any form that does not allow all four basic changes as constituents cannot represent all possible changes.  $\square$

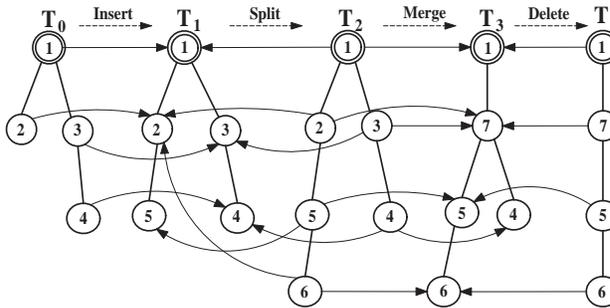


Figure 8. A complex change

Having proved that the normal form must include all types of basic changes, we now prove that the four types of basic changes must be structured in a particular order.

**Theorem 5** Any form, in which the changes are composed of the four basic changes but in a different sequence from the normal form, cannot represent all changes.

**Proof:** We prove this theorem by providing examples of changes that cannot be composed of the four basic changes in a different sequence to ISMD. Consider the three examples shown in Figure 9. (In the discussion, we use  $X_i$  to represent the component represented by the vertex  $i$ .)

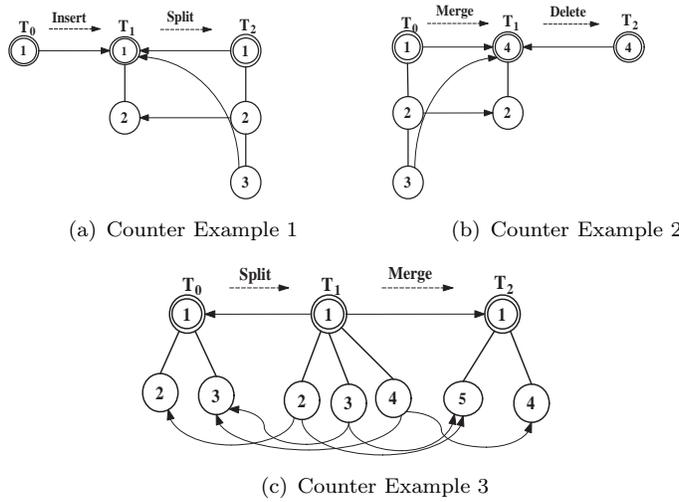


Figure 9. Counter Examples

Figure 9(a) shows a complex change composed of a basic insert and a basic split. This change starts from the state of a single negative component  $X_1$  (the whole spatial domain). During the change another negative component  $X_3$  is split from  $X_1$ . The split would never occur before a positive component exists to separate  $X_1$  and  $X_3$ . The positive component must be introduced by an insert. Thus, this change can never be equivalent to a complex change, in which there is no basic split after a basic insert. Hence any normal form, in which there is no basic split after a basic insert, cannot represent all changes.

Figure 9(b) shows a complex change composed of a basic merge and a basic delete. In this change, in order to separate components  $X_3$  and  $X_1$ , component  $X_2$  can never be deleted before the  $X_3$  is merged with  $X_1$ . Thus, this change can never be equivalent to a complex change, in which there is no basic delete after a basic merge. Hence any normal form, in which there is no basic delete after a basic merge, cannot represent all changes.

Figure 9(c) shows a complex change composed of a basic split and a basic merge. At the beginning of the change there are positive components  $X_2$  and  $X_3$ . During the change,  $X_3$  splits. Part of  $X_3$  merges with  $X_2$  and transforms to component  $X_5$ . The rest part of  $X_3$  transforms to  $X_4$ .  $X_3$  can never merge with  $X_2$  before  $X_3$  splits, otherwise it is unable to get  $X_4$  at the end of the change. Thus, this change could never be equivalent to a complex change, in which there is no basic merge after a basic split. Hence any normal form, in which there is no basic merge after a basic split, cannot represent all changes.

Based on the three examples, it follows that given any form, if it is composed of all four basic changes, and can represent any complex change, then the sequence of the four basic changes are determined as ISMD. This is the sequence of basic changes in the normal form.  $\square$

## 6. Conclusions and future work

In this paper, we have specified basic and complex changes of areal objects as tree morphisms. We also proposed a normal form that allows us to formally describe and compare spatial events according to changes in their topological structure. We are now able to revisit the examples presented in Figure 5. The change in Figure 5(a) is already in normal form, and therefore cannot be further simplified. The change in Figure 5(b) is not, and can be expressed by a simplified change in normal form, which is composed of a basic insert followed by a basic merge, shown in Figure 10.

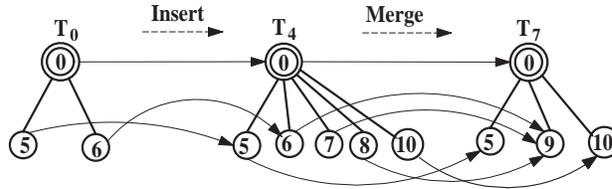


Figure 10. A complex change in normal form equivalent to the change in Figure 5(b)

The work presented here uses a single tree to represent the ‘surrounded by’ relation between components of areal objects. However, there are some topological properties that cannot be represented by a single tree. For example, our methods cannot differentiate certain cases where boundaries touch. For example, the two different splits in Figure 11, presented in (Galton 1997), are considered to be the same in our model.

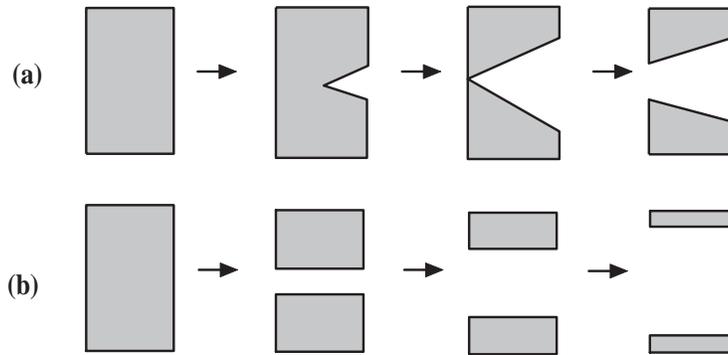


Figure 11. Two different splits (from (Galton 1997))

We are currently extending the tree model, so that richer topological structures and topological changes can be represented. One possible avenue for future research is to represent the topological structures by more than one tree. For example, trees representing adjacency relationships between region interiors and closures are capable together of capturing a richer collection of topological relationships.

Our work specifies topological changes based on the topological structures of areal objects. In order to determine the types of topological changes, sequential snapshots of areal objects are necessary. However, in many applications, it is somewhat difficult to capture such global information. Thus, it is also important to specify topological changes based on local, uncertain and incomplete information. Ongoing work by the authors aims at classifying the topological changes according to the information in a local area in which a change of areal object is observed. This theoretical research is currently been applied to an analysis of topological changes

captured as spatially-referenced time series by sensors in the Gulf of Maine (GoMOOS 2002).

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